NUMERICAL SIMULATION OF MARANGONI CONVECTION WITHIN HORIZONTAL TUBE FALLING FILM ABSORBERS

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ABSTRACT

The recent technical development of absorption systems pushes towards increasingly complex plant configurations, and, in fact, seems to stand a step forward with respect to the theoretical background needed for an accurate modelling of their performance. The main unresolved challenges in this respect are related to the film thickness and the extension of the phase-interfaces within falling film heat exchangers, which, in turn, are determined by the film wettabiity and interfacial turbulence effects such as surfactant-induced Marangoni convection. More specifically, the complex coupling between these aspects and the heat and mass transfer phenomena of falling film absorption is excluded from previous modelling approaches, which often fall short in reliably predicting the real performance of these devices.

As a result, this work presents an attempt to locally include the effect of Marangoni convection in the modelling of falling film absorption around horizontal cooled tubes, while considering the circular shape of the solid surface and the local thickness distribution. A two-dimensional model of the fundamental governing equations is formulated and a first screening of its numerical approximation results is presented. The generation of Marangoni cells responsible for the internal mixing and advection processes is highlighted. This leads to a continuous renovation of the interfacial conditions, intensifying the transport of the refrigerant, together with the related enthalpy, to the core of the solution. The present model can be applied to the analysis of advanced absorption systems for their development of optimal design and control strategies.

INTRODUCTION

In 1865 the Italian physicist Carlo Giovanni Marangoni studied for the first time the so called “Tears and wine” effect. Afterwards, the fluid flow resulting from surface tension differences, was categorized as Marangoni convection, studied and applied to many scientific and technical fields [1]. Although enhancement additives have been commonly used in LiBr absorption machines since the 1950s, the mechanism by which they improve heat and mass transfer rates, neither the limit to the enhancement nor the best additive for a certain absorptive mixture are currently known [2]. Kashwagi et al. [3] experimentally described the interfacial Marangoni instabilities within vapour absorption systems as mainly caused by the surface tension difference around immiscible surfactants islands in multi-components mixtures where the corresponding solubility limit was being overcome. Hozawa et al. [4] suggested heterogeneous absorption to be the natural cause of Marangoni convection and Hihara and Saito [5] studied this phenomenon in relation to falling film absorption. However, there is still not research covering the direct modelling of Marangoni convection within horizontal tube falling film absorbers. This work represents a first attempt to locally include this effect in the modelling of falling film absorption around horizontal cooled tubes, while considering the local thickness distribution and surface inclination [6] due to the circular shape of the solid surface.

A two-dimensional numerical model of the fundamental governing equations is formulated and a first screening of the results is presented. Different solution flowrates are considered to quantify local and global performance enhancement related to Marangoni convection in real absorbers. The generation of a Marangoni cells responsible for the internal mixing and advection processes is highlighted. This leads to a continuous renovation of the interfacial conditions, intensifying the transport of the refrigerant, together with the related enthalpy, to the core of the solution. The interdependences between heat, mass and momentum transfer are discussed in a way useful to clarify the process. The present model can be applied to the analysis of advanced absorption systems for their development of an optimal design and control strategies.

GOVERNING EQUATIONS

A schematic representation of the physical system is illustrated in Figure 1.

Figure 1. Representation of the system under analysis.
An incompressible Newtonian fluid, with constant thermo-physical properties (aside from surface tension, defined as a function of concentration and temperature), is laminarly flowing in a steady state on the solid surface of the tube, with no interfacial shear force and interfacial waves. Meanwhile, vapour absorption occurs at the liquid-vapour interface. The interfacial gas-phase resistance for heat and mass transfer is considered to be negligible, the absorptive solution at the interface is assumed to reach the equilibrium under the specific vapour pressure \( p \) and the solid wall temperature \( T_o \), remaining constant throughout the process. Furthermore, film interfacial deformations and viscous dissipation are disregarded. Body fitted coordinates (\( x \) along the tube surface and \( y \) normal to it at any point) are used because the film thickness is small if compared to the tube diameter. Surface tension gradients lead to a shear stress which affects the fluid motion in proximity of the interface and consequently the contingent mass transfer process. Marangoni convection is studied as imaginarily steady disturbances \( u' \), \( v' \) (Eq.1) to the hydrodynamics of the fully developed laminar film [5], which subsequently affects heat and mass transfer performance of the system under consideration.

\[
u(X,Y) = u_0(X,Y) + u(X,Y) \quad (1)
\]

The solution of the momentum equation for an undisturbed laminar film with defined thermo-physical properties leads to the well-known Nusselt integral solution \( u_0 \) (Eq. 2).

\[
u_0 = \frac{\rho g \delta^2 \sin \beta}{2 \mu} \left( \frac{\nu}{\delta} - \frac{\nu^2}{\delta^2} \right) \quad (2)
\]

Accordingly, once the film mass flowrate per unit length of the tube is known, the corresponding film thickness is given by Eq. (3).

\[
\delta = \left( \frac{3 \mu D}{\rho g \sin \beta} \right)^{1/4} \quad (3)
\]

Under the above stated assumptions the dimensionless expressions of the governing equations are described as follows. With reference to Lee et al. [7], dimensionless variables and coordinates are defined as,

\[
U = \frac{u \pi r \rho}{\mu} \quad (4)
\]

\[
V = \frac{v \pi r \rho}{\mu} \quad (5)
\]

\[
X = \frac{x}{\pi r} \quad (6)
\]

\[
Y = \frac{y}{\pi r} \quad (7)
\]

Continuity equation is consequently assured by means of Eq. (8).

\[
\frac{\partial U'}{\partial X} + \frac{\partial V'}{\partial Y} = 0 \quad (8)
\]

**Differential momentum transfer**

The momentum conservation in the stream-wise \( x \)-direction and in the normal \( y \)-direction is given by Eq. (9) and Eq. (10).

\[
U \frac{\partial U'}{\partial X} + V \frac{\partial U'}{\partial Y} = \frac{\partial}{\partial X} \left( \frac{\partial U'}{\partial X} + \frac{\partial}{\partial Y} \right) - \frac{\partial P}{\partial X} \quad (9)
\]

\[
U \frac{\partial V'}{\partial X} + V \frac{\partial V'}{\partial Y} = \frac{\partial}{\partial X} \left( \frac{\partial V'}{\partial X} + \frac{\partial}{\partial Y} \right) - \frac{\partial P}{\partial Y} \quad (10)
\]

**Differential heat and mass transfer**

The energy transport equation is written as in Eq. (11).

\[
U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{Pr} \left( \frac{\partial}{\partial Y} \left( \frac{\partial \theta}{\partial Y} \right) + \frac{\partial}{\partial X} \left( \frac{\partial \theta}{\partial X} \right) \right) \quad (11)
\]

where, \( Pr = \frac{\mu C}{k} \) \quad (12)

\[
\theta = \frac{T - T_w}{T_o - T_w} \quad (13)
\]

\( k \) is the thermal conductivity of the absorptive solution. \( T_o \) and \( T_w \) represent, respectively, the inlet stream and the solid wall temperatures. Lastly, the species transport equation is expressed by Eq. (14).

\[
U \frac{\partial \omega}{\partial X} + V \frac{\partial \omega}{\partial Y} = \frac{1}{Sc} \left( \frac{\partial}{\partial Y} \left( \frac{\partial \omega}{\partial Y} \right) + \frac{\partial}{\partial X} \left( \frac{\partial \omega}{\partial X} \right) \right) \quad (14)
\]

where,

\[
Sc = \frac{\mu}{\rho D} \quad (15)
\]

\[
\omega = \frac{C}{C_0} \quad (16)
\]

**Boundary conditions**

The fundamental equations governing the momentum, energy and species transport phenomena are solved with a finite difference approximation, while adopting a fully implicit numerical scheme in the calculation domain and
applying the following boundary conditions. At the solid wall \( Y=0, X \cap (0,L) \):

\[
U(X,0) = 0, \quad V(X,0) = 0, \quad \theta(X,0) = 0, \quad \frac{\partial \omega}{\partial Y}(X,0) = 0
\]

(17)

which correspond to the non-slip condition, constant temperature and impermeability of the wall to the LiBr species. At the inlet \( x=0, y \cap (0,\delta) \):

\[
U(0,Y) = U_b, \quad V(0,Y) = 0, \quad \theta(0,Y) = \theta_s, \quad \omega(0,Y) = \omega_b
\]

(18)

Marangoni convection can originate from interfacial surface tension gradients due to temperature gradients (thermo-capillarity) or both temperature and species concentration gradients (distill-capillarity). As in previous literature (e.g. Boyadjiev and Halatchev 1998 [8]) the surface tension gradient is balanced by the shear stress tensor \( t_r \).

\[
\tau_r(x,\delta) = \frac{\partial \sigma}{\partial x}(x,\delta)
\]

(19)

Under the assumption of a Newtonian fluid behaviour, and in a dimensionless expression,

\[
\frac{\partial U}{\partial Y}(X,\frac{\delta}{\pi r}) = \left( \frac{L \rho \sigma}{\mu r} \right) \frac{\partial \sigma^*}{\partial X}(X,\frac{\delta}{\pi r})
\]

(20)

Where, \( \sigma^* = \sigma/\sigma_0 \) is the normalised surface tension, and the definition of Marangoni number can be introduced with reference to the right-hand side of Eq. (19), where the characteristic length \( L \) is taken as the half outer-circumference of the tube \( \pi r \).

\[
Ma = \frac{\pi r \rho \sigma}{\mu^2}
\]

(21)

When absorption takes place in the presence of additives, the surface tension in such a system is a function of the temperature, solute concentration and the surfactant concentration. As a first screening of the model results it is supposed that the surface tension depends on temperature \( T \) and working fluid concentration \( \omega_s = \omega_s(\theta, \omega) \), then, the interfacial tension gradient along a direction \( x \) writes:

\[
\frac{\partial \sigma^*}{\partial X} = \frac{\partial \sigma^*}{\partial \theta} \frac{\partial \theta}{\partial X} + \frac{\partial \sigma^*}{\partial \omega} \frac{\partial \omega}{\partial X}
\]

(22)

And Marangoni number can be separated into two different contributions (Eq.s 23-24).

\[
Ma_t = \frac{\partial \sigma^*}{\partial \theta} \frac{\partial \theta}{\partial x} + \frac{\partial \sigma^*}{\partial \omega} \frac{\partial \omega}{\partial x}
\]

(23)

\[
Ma_u = \frac{\partial \sigma^*}{\partial \omega} \frac{\partial \omega}{\partial x}
\]

(24)

Moreover, a liquid–gas interfacial pressure difference at a certain point of the fluid free surface, \( \Delta p = p_r - p_s \), and surface tension are related by the Laplace equation:

\[
\frac{\partial p}{\partial x} = \frac{\partial^2 \sigma}{\partial x^2}
\]

(25)

For any \( \sigma \) variation, Eq. (25) emphasizes the generation of a non-zero fluid pressure gradient with respect to coordinate \( x \) (according to \( p_r = \text{const.} \)). Moreover, this result is consistent with the “le Chatelier” principle: the system acting in order to counteract the effect of the appearance of a surface tension gradient at interface (Isvoranu and Staicovici, 2004 [9]). Indeed, Eq. (25) shows that when the surface free energy is decreasing in order \( (\partial^2 \sigma/\partial x^2 < 0) \), or increasing in order that \( (\partial^2 \sigma/\partial x^2 > 0) \), a fluid movement will be generated towards positive or negative \( x \) coordinate, respectively in order to increase or decrease its surface free energy. As a result, in the present analysis, the boundary conditions described by Eq. (26-28) are applied at the film interface \( Y = \delta/\pi r, X \cap (0,L) \):

\[
V(X,\frac{\delta}{\pi r}) = 0
\]

(26)

\[
\frac{\partial U}{\partial Y}(X,\frac{\delta}{\pi r}) = Ma_t \frac{\partial T}{\partial X} + Ma_u \frac{\partial \omega}{\partial X}
\]

(27)

\[
\frac{\partial P}{\partial X}(X,\frac{\delta}{\pi r}) = \frac{\partial^2 \sigma}{\partial X^2} \approx Ma_t \frac{\partial^2 \theta}{\partial X^2} + Ma_u \frac{\partial^2 \omega}{\partial X^2}
\]

(28)

Which, in this order, assure the kinematic condition for a constant film thickness, the balance between shear stress and surface tension gradients from temperature/concentration variations, and preserve the specific minimum free energy of the liquid-vapour interface.

Equation (29) constitutes the energy balance of the heat of vapour absorption \( \Delta h_{abs} \) released at the interface and conducted through the film thickness to the cooled wall. Finally, the thermodynamic equilibrium at the interface is established with reference to Eq. (30).

\[
\frac{\partial \theta}{\partial Y}(X,\frac{\delta}{\pi r}) = -\Delta h_{abs} \rho D \frac{\partial \omega}{\partial Y}
\]

(29)

\[
\theta(X,\frac{\delta}{\pi r}) = f_{eq}(\omega_f, p)
\]

(30)

RESULTS
A first screening of the model results is performed under boundary and inlet conditions typical of a falling film absorber operating within a refrigeration machine (Table 1). Velocity field, temperature and concentration distributions are calculated locally and used to estimate the transfer performance of this device.

![Figure 1. Velocity field of the film at the reference conditions (Table 1).](image)

<table>
<thead>
<tr>
<th>Re</th>
<th>p</th>
<th>( \omega_{in} )</th>
<th>( T_{in} )</th>
<th>( T_{w} )</th>
<th>( r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>1.5 Pa</td>
<td>0.6</td>
<td>50 °C</td>
<td>40 °C</td>
<td>8 mm</td>
</tr>
</tbody>
</table>

Given the momentum transfer phenomena represented by figures 1-2, the related heat and mass transfer phenomena are represented by figures 3 and 4 (at the conditions in Table 1). The Marangoni convection cell at the entrance is responsible for a recirculation process and an increased diffusion and mass transfer processes, which are associated to a higher release of the heat of absorption, leading to an increased local heat transfer coefficient.

Nusselt and Shwerwood numbers are calculated with reference to the heat and mass transfer coefficients defined by Eq. (30).

\[
htc = \frac{k \frac{\partial T}{\partial y}}{T_{in} - T_{w}}, \quad mtc = -\frac{D \frac{\partial \omega}{\partial y}}{\omega_{in} \omega_{w} - \omega_{s}}
\]
The peak of the local mass transfer coefficient, observable in figure 4, is related to the negative (sustaining vapour absorption and internal diffusion) radial component of the Marangoni cell. The local temperature and concentration fields are illustrated in figure 5 at corresponding Reynolds as those used in figure 2.

Temperature and concentration fields, in turn, illustrate the temperature and concentration gradients responsible for the Marangoni effect.

As partial wetting effects are neglected, for a given inlet temperature and concentration at a fixed vessel vapour-pressure, a lower Reynolds number is associated to a lower film thickness and more intense heat and mass transfer. In Figure 5, it becomes obvious how the heat transfer to the tube worsens and the vapour penetration distance decreases as the flowrate grows larger.

CONCLUSIONS

This work constitutes a preliminary effort towards the objective of modelling the characteristics of velocity, temperature and concentration fields, due to Marangoni convection, on the related improvement of heat and mass transfer performance of falling film absorption within horizontal tube absorbers. A numerical model for predicting this effect within an absorptive falling film over a cooled horizontal tube has been presented and a first screening of its results has been performed, specifically, investigating the influence of different solution flowrates. The generation of Marangoni cells responsible for the internal mixing and advection processes is highlighted. This leads to a continuous renovation of the interfacial conditions, intensifying the transport of the refrigerant, together with the related enthalpy, to the core of the solution. The model can be extended to consider multi-components concentrations (absorptive salt and surfactants additive) and dynamic dependence of the surface tension on these conditions, to conduct dynamic analysis of intense Marangoni convection.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>LiBr mass fraction</td>
</tr>
<tr>
<td>C_p</td>
<td>Isobaric specific heat, J/kgK</td>
</tr>
<tr>
<td>D</td>
<td>Mass diffusivity, m^2/s</td>
</tr>
<tr>
<td>g</td>
<td>Gravity, m/s^2</td>
</tr>
<tr>
<td>h</td>
<td>Specific enthalpy, J/kg</td>
</tr>
<tr>
<td>htc</td>
<td>Heat transfer coefficient, W/m^2K</td>
</tr>
<tr>
<td>k</td>
<td>Thermal conductivity, W/mK</td>
</tr>
<tr>
<td>Ma</td>
<td>Marangoni Number</td>
</tr>
<tr>
<td>mtc</td>
<td>Mass transfer coefficient, m/h</td>
</tr>
<tr>
<td>Pr</td>
<td>Prandtl Number</td>
</tr>
<tr>
<td>r</td>
<td>Outer tube radius, m</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds Number</td>
</tr>
<tr>
<td>Sc</td>
<td>Schmidt Number</td>
</tr>
<tr>
<td>Sh</td>
<td>Sherwood Number</td>
</tr>
<tr>
<td>T</td>
<td>Temperature, K</td>
</tr>
<tr>
<td>u</td>
<td>Streamwise Velocity, m/s</td>
</tr>
<tr>
<td>v</td>
<td>Normal Velocity, m/s</td>
</tr>
<tr>
<td>U</td>
<td>Dimensionless streamwise velocity</td>
</tr>
<tr>
<td>V</td>
<td>Dimensionless normal velocity</td>
</tr>
<tr>
<td>x</td>
<td>Local tangential position, m</td>
</tr>
<tr>
<td>y</td>
<td>Local normal position, m</td>
</tr>
<tr>
<td>X</td>
<td>Dimensionless concentration distribution</td>
</tr>
<tr>
<td>Y</td>
<td>Dimensionless normal position</td>
</tr>
</tbody>
</table>

Greek symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma)</td>
<td>Surface tension, J/m^2</td>
</tr>
<tr>
<td>(\theta)</td>
<td>Dimensionless temperature distribution</td>
</tr>
</tbody>
</table>
\( \Gamma \) Mass flow rate per unit length, kg/sm
\( \delta \) Film Thickness, m
\( \mu \) Dynamic viscosity, Pas
\( \rho \) Density, kg/m\(^3\)
\( \omega \) Normalised mass fraction
\( \tau \) Shear stress, Pa

Subscripts
0 Inlet/Nusselt integral solution
abs Absorption
av Average
e Equilibrium
if Interface
T Temperature
v Vapour
w Wall
\( \omega \) Related to concentration
* Normalised

REFERENCES


