Basic Study on Solid-Liquid Phase Change Problem of Water around Heat Transfer Tubes
- Influence of tube ellipticity and initial water temperature on natural convention -

Qiang-Sheng WANG\(^1\), Koichi HIROSE\(^2\) and Takashi FUKUE\(^3\)

\(^1\) Department of Mechanical Engineering, Civil and Environmental Engineering, Graduate School of Iwate University, Morioka, Iwate 020-8551, Japan

\(^2, 3\) Department of Systems Innovation Engineering, Iwate University, Morioka, Iwate 020-8551, Japan

Abstract
This study describes the solid-liquid phase change process of water in cold thermal storage systems by using computational fluid dynamics analysis. This report especially focuses on natural convection phenomena around the heat transfer tubes mounted in the cold thermal storage container. The natural convection accelerates heat exchange between the water and the heat transfer tubes and this will affect the amount of the ice generation. Therefore, understanding the process of natural convection in the cold thermal storage container is important to optimize the inside structure of the container. As for the influence of the horizontal distance, it can promote the freezing fraction ratio when the horizontal distance becomes bigger at the same conditions. Moreover, We found that the freezing fraction ratio of the horizontal type was bigger than that in the vertical type at the same conditions and the influence of the arranged angle was relatively small compared with the horizontal distance.

Introduction
Recent years, global warming and crustal movement have become a major threat, as a countermeasure, the use of natural and clean energy are being taken seriously once again. Cold thermal heat storage is one of the apparatus that use the absorption and release of large latent heat when the PCM is freezing and thawing. Generally, the cold thermal heat storage system can be roughly divided into static and dynamic type. For dynamic type, the problem is that the device has to be made large for the existence of ice breaking and scraping unit. However, the static cold thermal storage system can meet the demands of economic efficiency, security and environment friendly.

There are many researches on the thermal storage system were reported. For example, Saito et al. clarified the effect of natural convection to the freezing interface shape around a horizontal circular tube\(^1\), Sasaguchi et al. calculated the influence of initial water temperature and tube wall temperature on the solidification process\(^2\), Torigoe et al. examined the effects of tube numbers and arrangement on the liquid phase change process around the horizontal circular tubes\(^3\). Moreover, Hirose et al. analyzed the characteristic of phase change phenomenon around the heat transfer tubes immersed in water by using the multi-domain model and the single-domain model\(^4,5\). T.J.Scanlon et al. investigated the melting-freezing problem in the rectangular container\(^6\) and Ktayama et al. analyzed for the freezing problem on Stefan's Problem by the method of incorporating the latent heat into the specific heat\(^7\). Recently, Sasaki et al. investigated the freezing phenomenon and bridging time around two horizontal circular tubes immersed in water by using the single-domain model\(^8\).

We have investigated the freezing phenomenon and bridging time\(^9\) in our last paper. And here, we focused on the influence by the tube shape and initial water temperature. Especially, the natural convection phenomena around the heat transfer tubes.

Analytical Method
We assumed that the tubes of the physical model were placed in a region that filled with water (PCM) and the coolant was flowing inside of the heat transfer tubes, and then the frozen layer of ice would generate around the heat transfer tubes. Fig.1 shows the physical model and coordinate system of the two elliptical tubes used. Dot-dash line in the figure shows the surrounding border of the calculation region and the broken line is a part connecting the upper and lower calculation areas. Moreover, the coordinate system is a Cartesian coordinate system with the Origin on the lower left of the whole calculation area.

We introduced the following assumptions in our analysis:

1. The flow is uncompress two-dimensional laminar flow.
2. Density of liquid can be changed only by buoyancy term.
3. There is no change in volume due to the phase change.
4. Super cooling phenomenon will not occur during the phase change.

![Fig.1 Physical model and coordinate systems for two elliptical tubes.](image-url)
**Fundamental Equations**

The fundamental equations in Cartesian coordinate system \((x, y)\) was converted to the general coordinate system \((\xi, \eta)\) by using the boundary fitted coordinate method. Fundamental equations of the two-dimensional Cartesian coordinate system are shown in the following:

**Continuity equation:**

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  

(1)

**Equation of motion in the x-direction:**

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\nu}{\rho} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)
\]  

(2)

**Equation of motion in the y-direction:**

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{\nu}{\rho} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + b_y
\]  

(3)

**Energy equation:**

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{\rho} \left( \frac{\partial q}{\partial x} + \frac{\partial q}{\partial y} \right) - \nu \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\]  

(4)

Where, \(b_y\) represents the volume force, \(a\) represents the thermal diffusivity and \(a = \lambda / \rho \cdot c_p\).

The converted basic equations by the variable \(f\) according to the general coordinate system are shown in the following:

\[
J \left( \frac{\partial f}{\partial \xi} + \frac{\partial f}{\partial \eta} \xi \right) + \frac{\partial f}{\partial \eta} \eta = -\frac{\partial q}{\partial \xi} + J \left( q_{11} f_{\xi} + q_{12} f_{\eta} \right)
\]  

(5)

Where, operators in the above equations defined as the following:

\[
q_{11} = \frac{\partial^2 q}{\partial \xi^2} + \frac{\partial^2 q}{\partial \eta^2}, \quad q_{12} = \frac{\partial q}{\partial \xi} \frac{\partial q}{\partial \eta} + \frac{\partial^2 q}{\partial \xi \partial \eta},
\]  

(6)

\[
J = \xi \eta - \eta \xi
\]  

(7)

Variable \(f, \Gamma, S\) in the above formulas correspond to the following:

**Equation of motion,**

\[
\Gamma = x \quad \text{X-direction:}
\]  

\[
f = u, \quad \Gamma = y, \quad S = -\frac{1}{\rho} \left( \xi \frac{\partial p}{\partial \xi} + \eta \frac{\partial p}{\partial \eta} \right)
\]  

(8)

**Y-direction:**

\[
f = v, \quad \Gamma = y, \quad S = \frac{1}{\rho} \left( \xi \frac{\partial p}{\partial \xi} + \eta \frac{\partial p}{\partial \eta} \right) + g \beta (T - T_\infty) f(T)
\]  

(9)

**Expression of energy:**

\[
f = T, \quad \Gamma = a, \quad S = 0
\]  

(10)

Analysis was carried out by applying to the SIMPLE \(^{(10)}\) method and using a centralized grid discretized by the finite volume method. Physical properties used in the analysis are described in heat transfer engineering materials \(^{(11)}\). Moreover, as for the relationship between water density and temperature, we used the Fujii’s equation \(^{(12)}\) as shown in Fig.2.

**Boundary Conditions**

As initial conditions in the analysis, liquid phase was assumed to be stationary at the water temperature of \(T = T_{\text{w}}\) and temperature of the tube wall as a boundary condition was set at \(T = T_{\text{w}}\) at the beginning. Additionally, the surface of heat transfer tube is at no slip condition as for the analysis of flow. Dot-dash line in the model diagram shown in Fig.1 represents the outer boundary of the computational domain and in this paper we also set it with no slip condition.

**Solutions of Phase Change**

In this analysis, we used the single-domain model to solve the phase change. Fig.3 shows the relationship between specific heat and temperature. We assumed that there is a phase change zone (PCZ) and the latent heat was incorporated to the specific heat at phase change zone. Related equations and incorporated specific heat are as the following:

\[
L = \int_{T_i}^{T_f} c_{\text{latent}} (T) dT
\]  

(11)

\[
c_{ph} = \frac{c_{p,s} + c_{p,l}}{2} + \frac{L}{\Delta T}
\]  

(12)

Where, \(c_{\text{latent}}\) represents the specific heat of the latent heat, \(c_{ph}\) represents the incorporated specific heat, \(c_{p,s}\) and \(c_{p,l}\) is the specific heat of the solid phase and liquid phases respectively.
Experimental Apparatus and Method

Fig. 4 shows the schematic diagram of the experimental apparatus. The test piece are two copper elliptical tubes. The ellipticity of the elliptical tube is 0.85. In addition, the horizontal type and vertical type were defined as the fig. 5. Fig. 6 are photographs of the test section.

The size of the test area is 270 mm in height, 180 mm in width and 170 mm in depth. Heat insulation material was covered around water tank in order to prevent the heat loss. T type thermocouples in a diameter of 0.2 mm were used to the measure the temperature of set points. Finally, photos of the solidification interface shape were taken at a predetermined time interval from the front side of test area.

Evaluation Method

The analysis and experimental results were evaluated by using the freezing fraction defined as the following:

\[
\text{Freezing fraction} = \frac{A_i}{A_0} \tag{13}
\]

Where, \(A_i\) and \(A_0\) represent the cross-sectional area of the heat transfer tube and the generated ice respectively.

Validity of the Present Numerical Analysis

In order to verify the validity of the present numerical analysis, we compared our analysis result with the exact solution (13) before bridging. Fig. 7 was the comparison of freezing fraction by exact solution and analysis. In this figure, solid line and plot points represent the exact solution result and analysis result respectively. We found that the freezing fractions in both cases were growing with time firstly very fast and then became slower. What’s more, the freezing fractions of them were matched very well.

![Comparison of freezing fraction by analysis and experiment](image)

Fig. 7 Comparison of freezing fraction by exact solution and analysis for two circular tubes before bridging. \((D_1 = 0 \text{ mm}, D_2 = 100 \text{ mm}, T_{in} = 0 \text{ °C}, T_{w,1} = T_{w,2} = -10 \text{ °C})\)

Fig. 8 showed the comparison of freezing fraction by analysis and experiment. Solid line represents the analysis result and plot points represent experimental result. We found that the result of analysis was a little higher than that of the experiment. The reason is that there existed heat loss more or less at experiment though we covered the heat insulation material around the water tank.

![Comparison of freezing fraction by analysis and experiment](image)

Fig. 8 Comparison of freezing fraction by analysis and experiment for two elliptical tubes. \((D_1 = 0 \text{ mm}, D_2 = 80 \text{ mm}, \phi_{g,1} = \phi_{g,2} = 90 \text{ °}, T_{in} = 4.0 \text{ °C}, T_{w,1} = T_{w,2} = -10 \text{ °C})\)
Fig. 10 are the temperature distribution of water tank in both analysis and experiment. As for the Upper, Middle and Lower key words in the items of figure 10, they represent the average temperature of all the grids in the upper part, two corresponding points on each sides at the middle part and all the grids in the lower part of the water container respectively which were shown as the black points in figure 9.

![Figure 9 Instructions of the temperature measurement points in both type of the water container.](image)

![Figure 10 Temperature distribution of water with time for freezing.](image)

![Figure 11 Comparison of freezing front contour by analysis and experiment for two elliptical tubes.](image)

We can see that the result of the upper side was higher in the experiment because of the heat loss. In addition, we also found that there was almost no temperature change with time and a relatively higher temperature of water was accumulating at the bottom side all over time. Moreover, the temperature of the middle and the upper sides were dropping over time rapidly. However, the average temperature of water in the upper side is dropping sharply between 3000 and 5000 seconds. This is because the cooling of the warm fluid in the upper part has completed and then the flow will be changed from downward flow to upward. And these factors will affect the freezing fraction in figure 11, we can see that the freezing fraction of upper tube was slight thicker compared with the lower tube both in the results of analysis and experiment.

**Results and Discussions.**

Figure 12 and fig. 13 are the typical results of isotherm and velocity fields when $T_{ini} = 4.0\, ^\circ\text{C}$ and $T_{ini} = 7.0\, ^\circ\text{C}$. And in both of the figures: (a) is 10 minutes after the analysis beginning; (b) is 30 minutes after the analysis beginning; (c) is just before bridging and (d) is immediately after bridging.

From fig. 12, we found that the temperature gradient near the bridged ice surface became small and thus produced a faster freezing area because it can absorb heat easily from the surroundings. Furthermore, the binding of ice completely separated the flow after bridging. Therefore, compared with the other part, temperature between the binding areas were lower so that the growth of ice could be higher in a short period immediately after bridging. Moreover, the rising flow was generating all over the time and then became weaker and weaker when the initial water temperature was 4 \, ^\circ\text{C}. However, as shown in fig. 13, downward flow generated firstly and then gradually transformed into the raising flow when the initial water temperature was 7 \, ^\circ\text{C}. This is because that water has the characteristic of density inversion and the higher density of water would firstly deposit at the bottom side of the water container when cooled at the initial water temperature of 7 \, ^\circ\text{C}. Therefore, the downward flow would transform into raising flow when the higher density of water full filled all the bottom side of the water container.
Fig. 12 Isotherms and velocity fields.

\( X = 180 \text{ mm}, Y = 270 \text{ mm}, D_x = 0 \text{ mm}, D_y = 80 \text{ mm}, \phi_{g1} = \phi_{g2} = 90^\circ, T_{\text{ini}} = 4.0^\circ \text{C}, T_{w,1} = T_{w,2} = -10^\circ \text{C} \) 

Fig. 13 Isotherms and velocity fields.

\( X = 180 \text{ mm}, Y = 270 \text{ mm}, D_x = 0 \text{ mm}, D_y = 80 \text{ mm}, \phi_{g1} = \phi_{g2} = 90^\circ, T_{\text{ini}} = 7.0^\circ \text{C}, T_{w,1} = T_{w,2} = -10^\circ \text{C} \)
Figure 14 shows the freezing fraction ratio by analysis in different horizontal distances and initial water temperatures. We can see that extension of the freezing fraction rate gradually slowed with time in both of the horizontal and vertical types when the initial water temperature was 4 °C. However, there is a recurring point at 7 °C in both cases. This is because the cooling of the warm fluid in the water container (as we can see from fig.10 (b)) used the energy that can make ice, thus produced a deceleration zone. In addition, the freezing fraction ratios are all growing with the increasing of horizontal distance.

Moreover, the freezing fraction ratio of horizontal type are relatively small, especially, in the case of the lower tube in the vertical type is relatively lower and the influence of arranged angle is smaller than that in the case of Dy = 0mm, Horizontal.

We took the average temperature of the upper, middle and lower parts of the water container to investigate the typical temperature distribution. Figure 16(a) is the average temperature distribution of the water container when Tini = 4 °C by analysis. As for the Upper, Middle and Lower key words in the items of figure 16, they represent the average temperature of all the grids in the upper part, two corresponding points on each sides at the middle part and all the grids in the lower part of the water container respectively shown as figure 9. We can see that the lower temperature of water container in the case of X = 270 mm, Y = 180 mm was cooled faster than that in the case of X = 180 mm, Y = 270 mm. This is because the lower tube could invade easier into the region in which deposited with higher density of water around 4 °C and taking small distance in the vertical direction of the water container. So, in the case of X = 270 mm, Y = 180 mm, the energy would be used for cooling the lower region instead of generating ice at first. As a result, the lower part of the container size X = 270 mm, Y = 180 mm is considered to be easily cooled compared to the water container size of X = 180 mm, Y = 270 mm. In addition, the upper and middle part temperature of X = 270 mm, Y = 180 mm are cooled slower than that in the case of X = 180 mm, Y = 270 mm.

Influence of the container size and initial water temperature

Figure 15 shows the freezing fraction ratio by analysis in different horizontal distances and tube-arranged angles. It can be seen that though the freezing fraction ratio are all growing with the increasing horizontal distance, the influence of arranged angle to the freezing fraction ratio is relatively small, especially, in the case of Tini = 7°C. Moreover, the freezing fraction ratio of horizontal type are bigger than that in the vertical type at the same conditions especially in the case of Tini = 4°C because the position of the lower tube in the vertical type is relatively lower and that means it will be take more energy to cool the 4 degree’s water. However, relatively speaking, the influence of arranged angle is smaller than the horizontal distance.

![Graph](image-url)
Figure 16(b) is the average temperature distribution of the water container when $T_{ini} = 7 \degree C$ by analysis. It can also be seen that the lower temperature of the water container in the case of $X = 270 \, mm$, $Y = 180 \, mm$ was cooled faster than that in the case of $X = 180 \, mm$, $Y = 270 \, mm$ as the same reason in the case of $T_{ini} = 4 \, ^\circ C$. From fig.15 (b), we can see that the average temperature of water in the upper side is dropping sharply between 2500 and 5000 seconds in both type of the water container. This is a switch from the downward flow to the upward flow because the cooling of the warm fluid that has stagnated in the upper part of the water container has been completed. Furthermore, the upper temperature of the water container is also cooled quickly than that the case of $X = 180 \, mm$, $Y = 270 \, mm$ because when the water container is wider and lower, the distance between the upper region and the upper tube will be nearer, so the cooled flows could reach the upper region quickly. However, it takes time to cool the upper region of the water container in the case of $X = 270 \, mm$, $Y = 180 \, mm$, because it is wider. Moreover, the influence of the initial water temperature has a similar trend and characteristics in the horizontal arranged type as described above according to the results of our research.

### Summaries

In this paper, we investigated the influences on freezing fraction ratio and natural convection by changing the aspect ratio of the water container and some other parameters of the elliptical heat transfer tubes. Moreover, we verified the rationality of the present model by doing experiments at the same conditions. Through the investigation, we obtained the following information and conclusions.

1. First, by comparing the results of analysis and experiment, we found that the qualitative features of the analysis were reasonable. Moreover, the analysis and experimental results matched very well. Especially, we captured the density inversion phenomenon when the initial water temperature is 7 $\degree C$. Therefore, we confirmed the validity of the present analysis.

2. As for the influence of the horizontal distance, it can promote the freezing fraction ratio when the horizontal distance becomes bigger at the same conditions. This is because the water around the heat transfer tube could be cooled and spread more quickly.

3. The arranged angle of the elliptical tube can also influence the freezing phenomena. We found that the freezing fraction ratio of the horizontal type was bigger than that in the vertical type at the same conditions. Especially in the case of $T_{ini} = 4 \, ^\circ C$, the lower tube can hardly be affected by the stratified lower region of the water container. However, the influence of arranged angle is relatively small compared with the horizontal distance.

### NOMENCLATURE

- $a$: Thermal diffusivity, $m^2/s$
- $A_c$: Cross sectional area of elliptical tube, $m^2$
- $A_t$: Cross sectional area of solidified layer, $m^2$
- $B (T)$: Density approximation function
- $c_p$: Specific heat, $kJ/(kg\cdot ^\circ C)$
- $D_i$: Horizontal distance of two tube centers, $mm$
- $D_v$: Vertical distance of two tube centers, $mm$
- $f$: General variable of $(\xi, \eta)$ coordinate system
- $g$: Gravitational acceleration, $m/s^2$
- $L$: Latent heat, $kJ/kg$
- $T_{wp}$: Freezing temperature of the PCM, $^\circ C$
- $T_{ini}$: Initial temperature of water, $^\circ C$
- $T_w$: Wall temperature of heat transfer tube, $^\circ C$
- $(u, v)$: Velocity of $x, y$ direction, $m/s$
- $(x, y)$: Cartesian coordinate system
- $U, V$: Velocity of $\xi, \eta$ direction in general coordinate system, $m/s$

### Greek symbols

- $\alpha$: Ellipticity
- $\beta$: Coefficient of thermal expansion, $1/K$
- $\Gamma$: Coefficient of diffusion item, $m^2/s$
- $\Delta T$: Temperature difference of phase change zone, $^\circ C$
- $\nu$: Coefficient of kinematic viscosity, $m^2/s$
- $\lambda$: Thermal conductivity, $W/(m\cdot ^\circ C)$
- $\rho$: Density, $kg/m^3$
\[ \phi \] Inclined angle of elliptical tube, °
\[ \xi, \eta \] General coordinate system by boundary fitted coordinate method

**Subscripts**
- \( l \) upper tube
- \( 2 \) lower tube
- \( l \) liquid phase
- \( s \) solid phase
- \( ph \) phase change
- \( t \) tube
- \( x \) x-direction differential, \( \partial / \partial x \)
- \( y \) y-direction differential, \( \partial / \partial y \)
- \( \eta \) \( \eta \)-direction differential, \( \partial / \partial \eta \)
- \( \xi \) \( \xi \)-direction differential, \( \partial / \partial \xi \)

**REFERENCES**